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Bohr’s inequality for harmonic mappings and beyond

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Abstract

© 2018, Springer Nature Singapore Pte Ltd. There has been a number of problems closely connected with the classical Bohr inequality for bounded analytic functions defined on the unit disk centered at the origin. Several extensions, generalizations and modifications of it are established by many researchers and they can be found in the literature, for example, in the multidimensional setting and in the case of the Dirichlet series, functional series, function spaces, etc. In this survey article, we mainly focus on the recent developments on this topic and in particular, we discuss new and sharp improvements on the classical Bohr inequality and on the Bohr inequality for harmonic functions.

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Keywords

Bohr radius, Bounded analytic functions, Rogosinski radius, Schwarz-Pick lemma, Subordination, Univalent functions

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